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**QCD, the Parton Model, and the Nucleon
Polarised Structure Functions**

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ABSTRACT

The present talk summarises the 1993 situation in understanding the spin structure of the nucleon via electron and muon polarised deep-inelastic scattering (PDIS). The central question I shall address here is if the data can be interpreted as evidence for polarisation in the “strange” nucleon “sea”, and I conclude that they can not: incidentally, I also find that they can not be constructed as evidence for violation of perturbative QCD (PQCD), either.

1. Introduction.

In April 1975, the cover of the CERN Courier represented the clamour raised by the then recent discoveries of the J/ψ and ψ' by two, radiative-tail-shaped piles of papers, the first and taller of which labeled “Theory”, the second, much lower and thinner, “Experiments”: such an image can very well represent again today what has happened to our community just after the 1988 publication by the EMC¹ of their results on the asymmetry in muon PDIS on a polarized hydrogen (but actually spin-frozen ammonia) target.

As then, these last few years have also seen subsequent experiments and deeper theoretical studies weeding out much of the “Theory” pile: in this, forcefully short, review, I shall concentrate only on the most significant, recent steps in this direction, and on their impact on data interpretation.

The rest of this paper will be divided in three parts: a presentation of the most advanced evaluations of PQCD corrections to the first-moment sum rules, a careful discussion of the original EMC experiment and of the problems posed in general when going from the asymmetry A to the polarized structure function g_1 , and then a discussion on the spin composition of the nucleon and in particular on its “strange” component.

2. PQCD developments.

2.1. The Bjorken sum rule.

The first-moment sum rule for the difference between proton and neutron polarized structure functions reads² (written in its full QCD garb)

$$\int_0^1 dx [g_1^p(x, Q^2) - g_1^n(x, Q^2)] = C(\alpha_s) \cdot \frac{1}{6} g_A + h.t. , \quad (1)$$

where $C(\alpha_s)$ are the PQCD corrections to the parton-model result, and “h.t.” stands for higher-twist contributions³ (HTC), behaving as integer inverse powers of Q^2 for $Q^2 \rightarrow \infty$. Since recent experiments extend from $\langle Q^2 \rangle = 2.0$ GeV² (SLAC E142⁴) to the 10.7 GeV² of EMC^{1,5}, inclusion of these effects might be crucial to a test of the Bjorken sum rule (BjSR), Eq. 1. This is indeed the point taken by Ellis and Karliner⁶ in their analisys of PDIS data.

An estimate of HTC has been given³ – and recently revised⁷ – by Balitskii, Braun and Kolesnichenko: two remarks are in order here. First, any evaluation of HTC to a sum rule such as Eq. 1 must depend on the order up to which $C(\alpha_s)$ is evaluated; second, the extrapolation away from $Q^2 = 0$ of the Drell–Hearn–Gerasimov sum rule⁸ suggests that HTC “die off” already at moderate values of Q^2 of the order of 1 GeV², much sooner thus than in the corresponding, “unpolarised” case of the Gottfried sum rule. This latter fact is substantiated by strong cancellation present – although between different-order terms – in the estimate of refs. (3,7).

I shall discuss in the rest of this sub-section the recent improvement⁹ in the evaluation of $C(\alpha_s)$ and its implications. This coefficient is now known to $O(\alpha_s^3)$ and can be written as

$$C(\alpha_s) = 1 - \frac{\alpha_s}{\pi} - c_1(N_f) \left(\frac{\alpha_s}{\pi} \right)^2 - c_2(N_f) \left(\frac{\alpha_s}{\pi} \right)^3 - \dots , \quad (2)$$

where the values of $c_{1,2}$ for N_f from 3 to 5 are listed in Table I, copied from the original paper by Larin and Vermaseren⁹. Due to the large values of the coefficients $c_{1,2}$, $C(\alpha_s)$ is evidently converging very slowly at low values of Q^2 , such as those of SLAC experiment E142⁴ and of the SMC¹⁰ ($\langle Q^2 \rangle = 4.6$ GeV²).

A technique for an estimate of the magnitude of higher, not yet computed terms is the so-called “accelerated convergence”, consisting in turning the power series into a continued fraction (or Padé approximant), for which convergence is rigorous only if the function expanded were a Stieltjes one (which is probably *not* the case for PQCD, but a test on the 2nd-order approximation to Eq. 2 shows that, despite this fact, the technique works reasonably well), namely in replacing Eq. 2 with

$$C(\alpha_s)^{-1} = 1 + \frac{\alpha_s}{\pi} [1 - [1 + c_1(N_f)] \frac{\alpha_s}{\pi} [1 - \frac{c_2(N_f) - c_1(N_f)^2}{1 + c_1(N_f)} \frac{\alpha_s}{\pi} [1 - \dots]^{-1}]^{-1}]^{-1} , \quad (2')$$

which one can expect to approximate the “full” $C(\alpha_s)$ better than Eq. 2 at the lowest values of Q^2 . Accordingly, one *must* use here the 3-loop expansion for $\alpha_s(Q^2)$ and its scale $\Lambda_{\overline{MS}}^{(N_f)}$: a recent PQCD analysis by Bethke and Catani¹¹ of all available data (both space-like and time-like) leads to a conservative estimate of $\Lambda_{\overline{MS}}^{(5)} = 200 \pm 50$ MeV, and therefore to the value $\Lambda_{\overline{MS}}^{(3)} = 411 \pm 103$ MeV that will be used throughout.

Table I
Coefficients of higher QCD corrections to BjSR

N_f	$c_1(N_f)$	$c_2(N_f)$
3	3.5833	20.2153
4	3.2500	13.8503
5	2.9167	7.8402

A question better addressed at this point is the actual value of N_f , the number of “active” flavours, to be used at each $< Q^2 >$: in annihilation, and in general for time-like Q^2 , there is no ambiguity, since flavour thresholds can be reasonably set for heavy quarks at $Q^2 \simeq 4m_Q^2$. In deep-inelastic scattering (DIS) a flavour is “active” *only* when appreciably contributing to the moment sum rules, i.e. when produced a) in a *really inclusive* manner (read: not only in low-multiplicity events), and b) over an appreciable range in Bjorken’s variable x (say up to $x \simeq 1/3$). If one sets the beginning of the scaling region at $Q^2 \simeq 2$ GeV² (as indicated by the “classic” SLAC-MIT experiments), the previous requirements ask for a $Q^2 \geq 17$ GeV² for charm to be an “active” flavour in DIS. The choice of $N_f = 3$ (rather than 4) has no great effect on $C(\alpha_s)$, as one can read from Table I, but for the Q^2 -evolution of the unitary-singlet piece the coefficient of the 1st-order term in Eq. 2 almost cancels, for $N_f = 4$, the 1st-order one, N_f/β_0 , coming from the anomalous dimension of the anomaly¹². The 1st-order formulæ used by Preparata and Ratcliffe¹³ are thus plainly wrong (not even mentioning their peculiar – to say the least – interpretation of the anomaly).

A quick summary of this sub-section is given in Table II, where we list the PQCD coefficients truncated at the m -th power for $m = 1$ and 3, $C(\alpha_s)_m$, together with the accelerated-convergence estimate $C(\alpha_s)_{ac}$, all evaluated with the 3-loop expression for $\alpha_s(Q^2)$ as in ref. 11: the reader can gauge by her/himself the impact of higher terms at the lowest Q^2 -values.

A comparison with the BjSR integral, as evaluated by ref. 6 and by Close and Roberts¹⁴, shows that HTC can indeed be only a minor correction at $Q^2 \geq 2$ GeV², provided one treats with adequate care the PQCD correction factor $C(\alpha_s)$ – or, if one likes it better, HTC can be easily traded for a slight modification of the latter,

such as provided by an “accelerated convergence” expansion.

Table II
The PQCD factor of BjSR

Q^2	α_s	$C(\alpha_s)_1$	$C(\alpha_s)_3$	$C(\alpha_s)_{ac}$
12.0	0.2395	0.9238	0.8940	0.8876
10.7	0.2454	0.9219	0.8904	0.8832
8.0	0.2619	0.9166	0.8800	0.8702
6.0	0.2809	0.9106	0.8675	0.8535
4.6	0.3018	0.9039	0.8529	0.8330
3.0	0.3445	0.8903	0.8206	0.7804
2.0	0.4015	0.8722	0.7714	0.6726

2.2. The isosinglet sum rule and the anomaly’s anomalous dimension.

In 1974, Ellis and Jaffe¹⁵, faced with the problem of how to use a sum rule akin to Eq. 1 with only hydrogen data (and polarized targets different from H₂ were over 18 years in the future), used parton–model ideas to derive a sum rule for the 1st moment of g_1^p alone; the QCD–corrected version of such a sum rule is a part of PQCD as fundamental as BjSR, and is best written for the isoscalar combination of PDIS structure functions as

$$\int_0^1 dx [g_1^p(x, Q^2) + g_1^n(x, Q^2)] = C(\alpha_s) \cdot [\frac{1}{18} g_8 + \frac{2}{9} g_0(Q^2)] + h.t. . \quad (3)$$

Additional complications with respect to the isovector one, Eq. 1, arise from the facts a) that the isoscalar axial charges of the nucleon are not directly measurable, and at least g_8 is indeed better known via flavour–symmetry arguments (*modulo* symmetry–breaking effects) than through experiment (elastic neutrino–nucleon or parity–violating electron–nucleon scattering), and b) that the unitary–singlet one g_0 couples to the gluonic fields via the axial anomaly and possesses therefore anomalous dimensions¹², so that its evolution with Q^2 is not exhausted by the PQCD factor $C(\alpha_s)$ and must be explicitly computed.

Since the coupling is scheme–dependent, this point has been the centre of a very heated theoretical debate. To cut a long history short, one can summarise it by saying that, in the conventional parton language where the masses of the partons are neglected with respect to the momentum scale Q , one can put¹⁶

$$g_0(Q^2) = \sum_{i=1}^{N_f} \Delta q_i - N_f \frac{\alpha_s}{2\pi} \Delta G(Q^2) , \quad (4)$$

where ΔG is the first moment of the gluon polarised distribution function $\delta G(x) = G_+(x) - G_-(x)$, and determine its evolution via the equation (where $t = \log Q^2/\mu^2$)

$$\frac{d}{dt}g_0(t) = -N_f \frac{\alpha_s}{2\pi} \gamma_{gq}(\alpha_s) g_0(t) , \quad (5)$$

which relates to the anomalous dimension of the axial anomaly $\gamma_{gg}(\alpha_s)$ via

$$-N_f \frac{\alpha_s}{2\pi} \gamma_{gq}(\alpha_s) = \gamma_{gg}(\alpha_s) - \beta(\alpha_s) \frac{2\pi}{\alpha_s} \quad (6)$$

and gives, after integrating in α_s ,

$$\begin{aligned} \log \frac{g_0(Q^2)}{g_0(\mu^2)} &= \frac{6N_f}{33 - 2N_f} \frac{\alpha_s(Q^2) - \alpha_s(\mu^2)}{\pi} [1 + \\ &+ \left(\frac{83}{24} + \frac{N_f}{36} - \frac{33 - 2N_f}{8(153 - 19N_f)} \right) \frac{\alpha_s(Q^2) + \alpha_s(\mu^2)}{\pi} + \dots] , \end{aligned} \quad (7)$$

with the 3-loop calculation by Larin¹⁷ of the anomalous dimension.

As one can read from the above expression, $g_0(Q^2)$ can be drastically reduced (still at $N_f = 3$) from its value at the scale μ^2 for $Q^2 > \mu^2$; on the other hand, one can not set $\mu \rightarrow \infty$ and thus drop $\alpha_s(\mu^2)$ from Eq. 7, since the anomaly contribution to Eq. 4 is not definable in this limit¹⁶ due to $\Delta G(Q^2)$ growing asymptotically as $\alpha_s(Q^2)^{-1}$. It can also be noted that $g_0 = 0$ is a special value, being a *fixed point* in the evolution equation¹⁶, but on the other hand this does not allow to infer, for the same reason that gives the evolution in Eq. 7, that $\sum_i \Delta q_i = 0$!

Sometimes, it has also been stated in the literature that the anomaly contributes to $g_1(x)$ only at very small x values: this is true (in the above, $m_i = 0$ scheme) *only if* the polarised gluon distribution peaks at $x = 0$, as e.g. in the “intrinsic” gluon distribution derived by Brodsky and Schmidt¹⁸. Unfortunately, the distribution they propose for $\delta G(x)$ has the wrong Regge behaviour for $x \rightarrow 0$ (*contrary to their statement*), since it would require dominance of δG by the *pion trajectory* with intercept $\alpha_\pi(0) \simeq 0$; instead, one would rather expect it to be dominated by the pseudoscalar–glueball trajectory with intercept $\alpha_G(0) \leq -1$. When this constraint is imposed on the Brodsky–Schmidt framework, *ceteris paribus*, more than 80% of the anomaly contribution falls in the x interval covered e.g. by the EMC experiment, making the anomaly contribution virtually indistinguishable from the other, “intrinsic”, “sea” distributions, while the integral ΔG remains of order unity, as in ref. 17, or less, being rather solidly tied to the momentum fraction $\langle x_G \rangle \simeq 1/2$ carried by the gluons at the momentum scale at which these “intrinsic” components are defined.

3. Measurements and parametrizations for $g_1(x)$.

What is actually measured are not the PDIS structure functions themselves, but rather the polarisation asymmetries $A = (\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow})/(\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow})$, related to the former by

$$g_1(x, Q^2) = \frac{A \cdot F_2(x, Q^2)}{2x \cdot [1 + R(x, Q^2)]}. \quad (8)$$

It is obvious that the factor $2x$ in the denominator makes the direct determination of $g_1(x)$ at $x \rightarrow 0$ impossible for finite-accuracy data. The evaluation of the sum rules, Eqs. 1 and 3, depends thus on the parametrisation assumed to extrapolate $g_1(x)$ to $x = 0$: the usual treatments of this point have till now assumed it to extrapolate *smoothly* to a constant as $g_1(x) \simeq \alpha + \beta x$, in accord with the pion-pole trajectory intercept being close to zero. However, since one does not expect the “sea” distributions to couple dominantly to an isovector, pseudoscalar trajectory, such as the pion, but rather to an isoscalar one such as the eta, I shall rather have for them a behaviour $x^{-\alpha_\eta(0)}$, with $\alpha_\eta(0) \simeq 1/4$, which, together with the negative sign of the “sea” contribution, produces a spike *in the isoscalar part* of $g_1(x)$ at $x = 0$ of the type $g_1(x) \simeq \alpha - \beta x^{-1/4}$ (with $\alpha, \beta > 0$), presently unseen in the asymmetry A just because of the $2x$ factor and the very *limited* accuracy of the data. Of course, such a “spiky” behaviour does not show in the integrand of the BjSR, which can therefore be extrapolated smoothly to $x = 0$ according to the conventional practice in this matter.

Apart from this point, which has however a non-negligible influence on the evaluation of the isoscalar sum rule, Eq. 3, raising the low- x contribution to its l.h.s. well over the “conventional” estimates, one has to avoid using the EMC published data⁵ for g_1^p , and to use instead only their values for A , with an adequate set of values for F_2 and R .

Indeed, the EMC calculated originally¹ g_1^p using a) the ratio R predicted by PQCD, systematically smaller than experiment since finite-mass corrections are dominant at low values of Q^2 (the effect of this is however not too important at $Q^2 = 10.7$ GeV²), and b) *their* values for the *unpolarized* structure function F_2^p , systematically lower than those by the BCDMS collaboration¹⁹ (and than the recent NMC data²⁰ as well) by as much as 13% at the lowest values of x .

Even using their measured values of A together with a phenomenological parametrization for $F_2^p(x, Q^2)$ (and $R^p(x, Q^2)$) to produce $g_1^p(x, Q^2)$ at a reference, fixed value of Q^2 (and assuming A to vary little^{6,14} with Q^2 –a yet to be proven assumption, in the light of the sparse nature of available data) is not free of the above, last source of error: indeed only the latest, post-NMC parametrizations²¹ have dropped the *unpolarized* EMC data altogether, while *all* previous analyses ended up averaging over the two, conflicting sets of data for small values of x , as did the EMC in the full-paper version of their work⁵.

4. The isosinglet sum rule and the spin content of the nucleon.

In the following table we present a re-evaluation of the integrals over the three experiments^{4,5,10} on the PDIS structure functions g_1 , with a personal renormalization of the EMC $g_1^p(x)$ data, following the prescriptions outlined in the previous section. There is an evident increase in the proton integral over the EMC evaluation of ref. 5: the same behaviour at $x = 0$ has been assumed for all three PDIS structure functions, and the same increase is present for them as well for the low- x part of the integrals. Due to the different nature of the targets and momentum scales we correct each integral with the appropriate BjSR contribution, where we use the $C(\alpha_s)$ given by “accelerated convergence” (Eq. 2), and the value $g_A = 1.2555 \pm 0.0015$ from an overall analysis of all baryon semileptonic decay data²².

Table III
The isosinglet sum rule evaluation and analysis

Experiment	revised EMC ⁵	SMC ¹⁰	SLAC E142 ⁴
data x -range	0.01 – 0.70	0.04 – 0.070 [†]	0.03 – 0.70
from data	0.129 ± 0.013	0.039 ± 0.023	-0.019 ± 0.008
from low- x	0.006 ± 0.001	0.005 ± 0.003	-0.005 ± 0.002
from high- x	0.001	0.001	0.000
$\int_0^1 dx g_1$	0.136 ± 0.013	0.045 ± 0.023	-0.024 ± 0.008
BjSR	0.1848 ± 0.0002	0.1743 ± 0.0002	0.1407 ± 0.0002
isoscalar SR	0.087 ± 0.026	0.090 ± 0.046	0.093 ± 0.016
$g_0(\alpha_s)/g_0(1)$	0.6820	0.6953	0.7220
$g_0(1)$ for $g_8 = 0.75$	0.376 ± 0.194	0.430 ± 0.358	0.602 ± 0.149
$g_0(1)$ for $g_8 = 0.60$	0.431 ± 0.194	0.484 ± 0.358	0.654 ± 0.149

[†]) range reduced to avoid screening effects on the very-low- x data points.

Here we derive $g_0(\alpha_s = 1)$ either using the flavour $SU(3)$ prediction $g_8 = 3g_A^{\Xi\Lambda} \simeq 0.75$ or the perhaps better value $g_8 \simeq 0.60$, which includes a simple modeling of $SU(3)$ -symmetry-breaking effects in the baryon octet²³ (reproducing the increase in $\Sigma_{\pi N}$ over the quark-model prediction without a large “strange sea” contribution²⁴).

The reduction of $g_0(\alpha_s = 1)$ with respect to g_8 can be explained by the gluonic anomaly without any recourse to a “strange” component in the spin density: for $g_8 = 0.750$ we expect indeed $g_0(\alpha_s = 1) = g_8 - \frac{3}{4\pi}(1 - g_8) = 0.690$ (in somewhat poor agreement with line 9 of Table III), and, for the smaller value $g_8 = 0.600$, $g_0(\alpha_s = 1) = 0.505$, in good accord with the results of the last line in the table, and leading to a rather small “intrinsic” value for $\Delta G(\alpha_s = 1)$.

4. Summary and conclusions.

The three experiments on the PDIS asymmetries (SLAC experiments E142⁴, and the collaborations EMC^{1,5} and SMC¹⁰ at CERN) do not contradict conventional expectations on the spin structure of the nucleon, namely that one should have a “strange” spin component very close to zero on one side, but on the other no pure valence–quarks either, as known since more than twenty years (and a complete list of references would be as long as this paper itself: I just refer to the recent, illuminating papers by Lipkin²⁵) from the reduction in g_A with respect to the quark–model prediction $g_A = 5/3$.

One finds a drastic reduction in $g_0(Q^2)$ from the parton–model–plus–OZI–rule expectation $g_0 \simeq g_8$, due *both* to the presence of the QCD axial anomaly and to its anomalous dimension (and, as one can read from Table III, due more to the second than to the first reason): a correct use of QCD with high orders included (which make these effects even larger than 1st–order alone) is necessary to describe the first–moment sum rules without conflict with experiments (and our expectations). The “spin crisis” of 1988 was the result of inadequate theoretical description, as much as of a somewhat low normalization in the F_2 values used by the EMC.

Last but not least, HTC are not needed to explain the data, at least for $Q^2 \geq 2 \text{ GeV}^2$ (the SLAC E142 average momentum scale), though essential in connecting the BjSR, Eq. 1, to the Drell–Hearn–Gerasimov sum rule at $Q^2 = 0$.

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